Radio Frequency Transformers using L-C Networks

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A FREQUENT requirement in radio frequency circuits is the ability to transform some particular value of resistance to some other value, without introducing a significant power loss in the process. An example of this is the matching of the effective r.f. anode load of a valve into the grid of the succeeding stage. One of the easist ways of

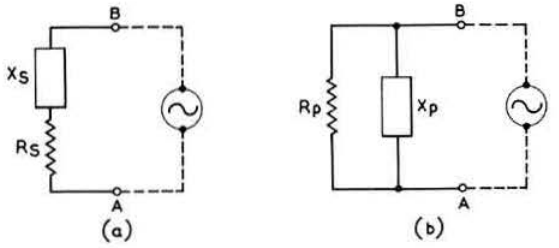

Fig. 1. Equivalent impedance circuits.

achieving this transformation is by the use of inductance and capacitance in a single network, using entirely reactive elements in the transformer. Clearly if such elements are perfect, then the power losses will be zero and the transformation will occur at unity efficiency. Since in practice, radio frequency inductors and capacitors can be made to a relatively high quality or high-Q, the losses in a practical arrangement are quite small, resulting only from the dissipation in the d.c. resistance of the coil due to the passage of current, and the dielectric loss in the capacitor due to the leakage current, which is in turn proportional to the voltage across it.

Consider two elementary networks in series and shunt elements respectively as shown in Fig. 1. Now for these two networks to be identical, that is to present the same load to a generator connected across the terminals AB, the admittance seen at these terminals must be the same in each case.

$$\frac{1}{Rp} + \frac{1}{jXp} = \frac{1}{Rs + jXs} \qquad \ldots . \text{(i)}$$

From this it is simple to derive the following two identities (Appendix 1) which are the key to all network problems:

$$Rp = Rs \left(1 + \frac{Xs^2}{Rs^2}\right) \qquad \ldots . \text{(ii)}$$

$$Xp = Xs \left(1 + \frac{Rs^2}{Xs^2}\right) \qquad \ldots \text{(iii)}$$

Using these basic formulae it is possible to convert a network of series elements to its equivalent network of parallel elements (and conversely). Consider next the circuit shown in Fig. 2. From an inspection of Fig. 1 and equation (ii) above, it can be seen that it is possible to select a value of Xs to put in series with the resistance Rs, such that the shunt equivalent resistance is equal to Rp, but has in parallel with it a residual reactance Xp. If now a reactance of value $-Xp$ is placed across the terminals AB, the net effect will be to produce at those terminals a pure resistance equal to Rp, which is the requirement of the transformer. Then the basic form of the transformer is an L network comprising only reactive elements as shown in Fig. 3. This also meets the terms of the original specification. If the ratio $R_2/R_1 = p$, then it can be shown (Appendix 2) that:

$$X_1 = \pm R_1\sqrt{(p-1)} \qquad \ldots \text{(iv)}$$

$$X_2 = \mp pR_1/\sqrt{(p-1)} \qquad \ldots \text{(v)}$$

Then X_1 and X_2 must have opposite signs in all cases, or in more general terms, the network must consist of one inductive and one capacitive element. It is more usual in practice to make the series element inductive and the shunt element capacitive for reasons which vary with individual circumstances (e.g. a series d.c. path to the anode of a valve), but there is no basic objection to the reversal of this practice in cases where it may be more suitable.

Using the fundamental equations (iv) and (v) it is now

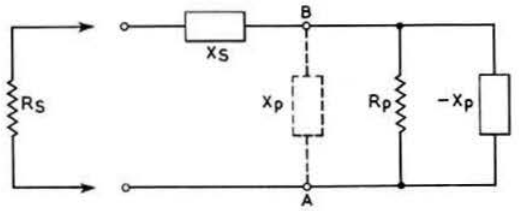

Fig. 2. Transformation : stage 1.

possible to design any required network by a process of arithmetic as the following example will show:

Example

A single wire short aerial for 3·6 Mc/s has a base impedance of 15-j200 ohms. It is required to match this to a feeder with a characteristic impedance of 75 ohms (Fig. 4 (a)). This aerial is equivalent at its terminals to a resistance of 15 ohm in series with a capitance of 220 pF. The steps of the calculation are as follows:

(a) Connect an equal and opposite reactance $+j200$ in series to tune out the aerial capacity and leave only the resistive term of 15 ohms: the transformation required is then from 15 ohms resistive to 75 ohms resistive (Fig. 4 (b)).

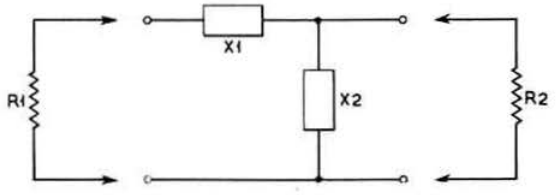

Fig. 3. Transformation : stage 2.

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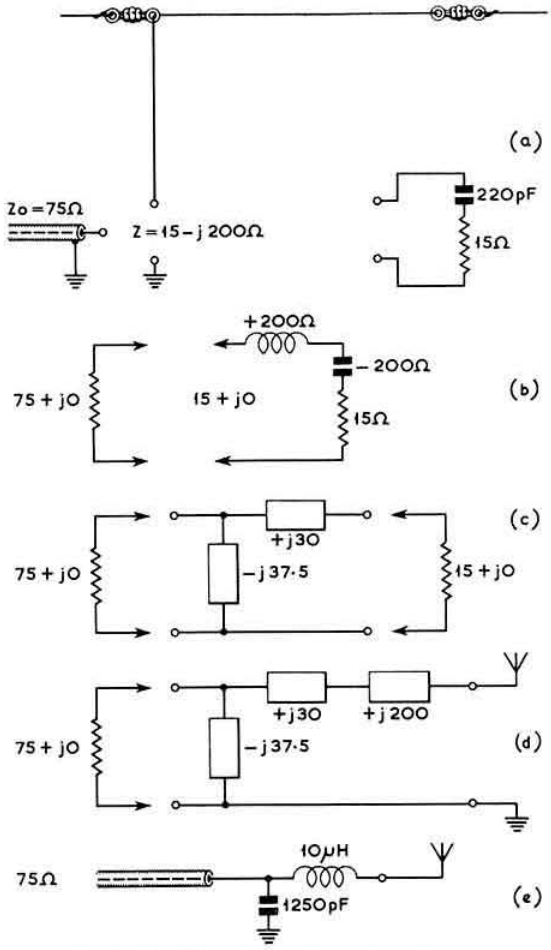


Fig. 4. Matching of two arbitrary impedances.

(b) From equation (iv), X_1 (series term) $= \pm 15\sqrt{5-1}$
$= \pm 30$ ohm

(c) From equation (iv), X_2 (shunt term) $= \mp \dfrac{5 \times 15}{\sqrt{5-1}}$
$= \mp 37{\cdot}5$ ohm

Thus the transformer section required is as shown in Fig. 4 (c) and the complete arrangement in Fig. 4 (d). To convert the reactance into physical components, since $X_L = 2\pi fL$ and $Xc = 1/2\pi fC$:

$+ j230$ is the reactance of 10 microhenries at 3·6 Mc/s.

$- j37{\cdot}5$ is the reactance of 1250 pF.

The complete network is then shown in Fig. 4 (e). By making each element variable over a convenient range, it is possible to correct for any inaccuracies in the original figure taken for the aerial base impedance, and also for variations in the working frequency.

A more complicated example of the use of *L-C* transformers is to be found in the familiar pi-coupler anode circuit encountered in transmitters. This is essentially a transformer of variable ratio, which can be used to adjust over a wide range the coupling between the valve anode and a fixed load represented by the aerial. The latter may be the actual aerial itself, although it is better practice to provide a permanent aerial coupler (or transformer) at the base of the aerial downlead to reduce the aerial feed impedance to 75 ohms resistive, and to connect this coupler to the output socket of the transmitter with 75 ohm co-axial cable. This has two immediate advantages in favour of TVI suppression:

(i) the connection between the transmitter and the radiating aerial proper is an unbalanced low impedance, i.e., not only is it in an earthed screened cable, but due to its low impedance the voltages and hence the radiated field will inherently be low.

(ii) low-pass TVI filters can be inserted in the co-axial line, and will be well terminated each end by 75 ohms, which is a necessary factor for the efficient use of such filters.

The pi-tank itself may be considered as two transforming networks back to back as shown in Fig. 5. In practice

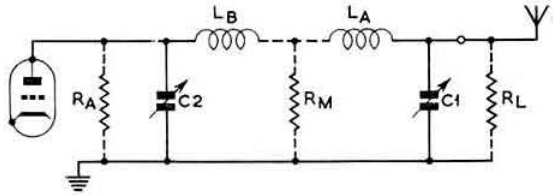


Fig. 5. Theoretical pi-coupled amplifier.

L_A and L_B are coalesced into one single coil. The " loading " capacitor *C1* is usually a variable condenser of fairly high maximum value: the reason for this is clear from an inspection of equations (iv) and (v). For a fixed value of load R_L, it will be seen that, as the reactance of the shunt element *C1* is reduced, the value of the mid-shunt resistance R_M also falls. This is equivalent to saying that for a low value of X_1 (i.e. a large value of *C1*), the value of R_M approaches zero or a short circuit, and to all intents and purposes the value anode tank circuit becomes L_B and *C2* in parallel, i.e. virtually no load resistance R_A appears in circuit, and the stage is completely uncoupled. Now as *C1* is decreased in value, R_M increased in value by the transformer action of *C1* and L_A, and consequently R_A increases in value, due to the upwards transformation of *C2* and L_B, and an increasing load is reflected into the value anode. As the value of R_M changes from equation (v), the value of *C2* must be adjusted to obtain the maximum value of R_A, i.e. this is equivalent to the " tuning " of a

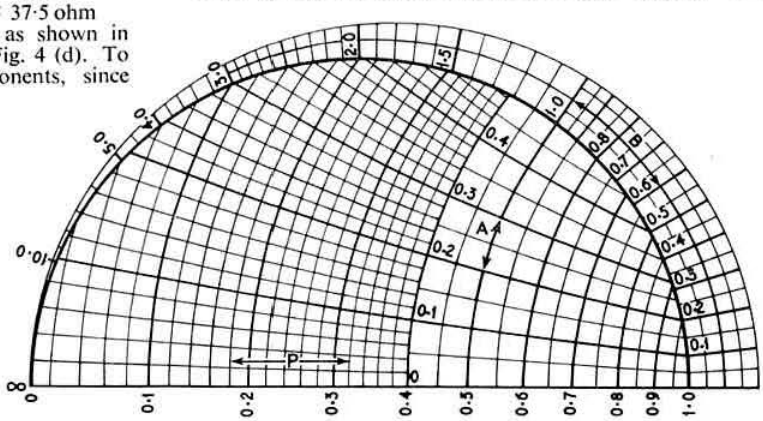
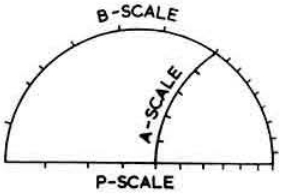


Chart for the design of L-C matching networks.

conventional parallel tuned circuit anode tank. Obviously the values of $C1$, $C2$ and $L\,(=L_A+L_B)$ are quite critical for optimum performance of a given network and it is a laborious business to use the arithmetical calculation for the resistance transformations involved. However it is possible to develop a simple chart from the Smith polar



Identification of Scales A, B and P.

impedance diagram which enables the calculations to be carried out more easily, and also enables the effect of changes in one of the variables to be rapidly assessed by inspection, which is not always obvious from the formulae.

The chart is illustrated at the foot of page 525, and the following simple rules describe its use.

To Match R1 and R2 using L and C

(i) Derive $p = R1/R_2$ (or R_2/R_1) such that p is less than 1.

(ii) Move from this point on the p axis along a constant A line to the edge of the semi-circle.

(iii) Note the A and B scale readings of this point.

If $R1 > R2$: X_L in series with $R2 = R1 \times A$
X_C in parallel with $R1 = R1 \div B$

If $R1 < R2$: X_L in series with $R1 = R1 \times B$
X_C in parallel with $R2 = R1 \div A$

Note that the shunt reactance X_C is *always* in parallel with the larger resistance.

Using the example of the 3·5 Mc/s aerial as before:
$R1 = 15 \quad R2 = 75 \therefore p = 0{\cdot}2 \therefore A = 0{\cdot}4 \quad B = 2{\cdot}0 \therefore X_L =$ 30 ohms and $X_C = 37{\cdot}5$ ohms.

Bibliography

[1] " H.F. Aerials," *Amateur Radio Handbook*, p. 368.
[2] " Simplified Design Procedures for Pi-Network Tank Circuits," G. C. Fox, R.S.G.B. BULLETIN, June 1961.
[3] " Mobile Antennas," *A.R.R.L. Antenna Handbook*, p. 283.
[4] " Matching with the L-C Network," *A.R.R.L. Mobile Handbook*, p. 270.

Appendix 1

For Fig. 1 (a) and Fig. 1 (b) to be identical at terminals AB:

$$\frac{1}{Rp}+\frac{1}{jXp}=\frac{1}{Rs+jXs} \qquad \ldots\ldots \text{(i)}$$

$$=\frac{Rs-jXs}{(Rs+jXs)\,(Rs-jXs)}$$

$$=\frac{Rs-jXs}{Rs^2+Xs^2}=\frac{Rs}{Rs^2+Xs^2}-\frac{jXs}{Rs^2+X^2}$$

Equating real and imaginary parts:

$$\frac{1}{Rp}=\frac{Rs}{Rs^2+Xs^2} \qquad \frac{1}{Xp}=\frac{Xs}{Rs^2+Xs^2}$$

$$\therefore Rp=\frac{Rs^2+Xs^2}{Rs}=Rs\left(1+\frac{Xs^2}{Rs^2}\right) \qquad \ldots\ldots \text{(ii)}$$

$$Xp=\frac{Rs^2+Xs^2}{Xs}=Xs\left(1+\frac{Rs^2}{Xs^2}\right) \qquad \ldots\ldots \text{(iii)}$$

Appendix 2

From equations (ii) and (iii)—Appendix 1:

$$Rp=\frac{Rs^2+Xs^2}{Rs} \qquad Xp=\frac{Rs^2+Xs^2}{Xs}$$

Then $Xs^2=(RpRs-Rs^2)=Rs^2\left(\frac{Rp}{Rs}-1\right)$

If $\frac{Rp}{Rs}=p$ then $Xs=\pm\,Rs\sqrt{p-1}$ $\qquad \ldots\ldots \text{(iv)}$

Also substituting for Xs:

$$Xp=\frac{Rs^2+(RpRs-Rs^2)}{\pm\sqrt{(RpRs-Rs^2)}}=\frac{RpRs}{\pm\sqrt{(RpRs-Rs^2)}}$$

$$=\frac{RpRs}{Rs\pm\sqrt{(p-1)}}=\frac{Rp}{\pm\sqrt{(p-1)}}=\frac{pRs}{\pm\sqrt{(p-1)}}$$

Then $Xp=\dfrac{\mp pRs}{\sqrt{p-1}}$ $\qquad \ldots\ldots \text{(v)}$

Round Britain Quiz

IN the B.B.C. All Britain Quiz broadcast on April 11, 1962, the London team was asked, " If I am an XYL listening to my OM sending 73 to GI and GM what am I? " It says much for Amateur Radio that the team gave a correct answer to the question immediately it was asked.

The question was sent in by Mrs. Mier, wife of Mr. P. D. Mier (G3KKK) of Hertford.

Current Comment **(Continued from page 523)**

capital loss which would result from the sale of these investments at the present time would be avoided. It is known that the rate of interest on a scheme of this kind would be 1 per cent over Bank Rate with a minimum of 6½ per cent.

In considering the cost of establishing new Headquarters it must be borne in mind that the rent being paid at present for the top floor at New Ruskin House amounts to £550 a year, and when the lease expires in 1963 it is possible that this figure may be increased. At the present time the total income received from investments, less the rent paid, leaves a net amount of about £100 a year in hand. Assuming the interest on the mortgage of £15,000 averaged 7 per cent throughout the 15 years—the amount borrowed being reduced as each security matured—and taking into account the amount of income from investments remaining, the difference would be an average net cost to the Society of about £350 p.a. At the end of the 15 years the mortgage would have been paid off and the Society would be in possession of freehold premises in place of investments. It must be remembered, however, that the Society would be directly responsible for the maintenance and upkeep of its own premises. This then is the position as it is today.

Those who have made a donation to Headquarters Fund have already been thanked by the President. This opportunity is now taken of thanking them on behalf of all members for their generosity. Those who have not yet made a contribution may rest assured that their support will be no less appreciated.

One final point: since the appeal was launched last September, approaches have been made directly to the Nuffield Trust for the Forces of the Crown and the Royal Commission for the 1851 Exhibition and unofficially to the Ministry of Education with a view to obtaining grants or donations, but so far the results have been disappointing. It would be of great assistance to the Council if any member has knowledge of a trust which might be in a position to make a grant to the Society. N. C.